

Dynamical probe of thermodynamical properties in three-dimensional hairy AdS black holes

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(Dated: November 26, 2014)

We study the quasinormal modes (QNM) of electromagnetic perturbations for rotating black hole with a nonminimally coupled scalar field hair and Banados-Teitelboim-Zanelli (BTZ) in three-dimensional anti-de Sitter (AdS) spacetimes. We find that the imaginary parts of quasinormal frequencies of dynamical perturbations can reflect the properties of thermodynamical stability disclosed by examining the free energy. In addition, we observe that the imaginary part of QNM frequencies can also determine the dynamically preferred stable configuration, which agrees with the study of the thermodynamical stability. This dramatic relation between dynamics and thermodynamics is also disclosed in three-dimensional static black hole backgrounds. The obtained results further support that the QNM can be a dynamic probe of the thermodynamic properties in black holes.

PACS numbers: 04.70.-s, 04.30.Nk, 04.25.-g, 04.70.Dy

I. INTRODUCTION

In black hole physics there is a famous no hair theorem. In general, the no-hair theorem rules out four-dimensional black holes coupled to scalar field in asymptotically flat spacetimes [1, 2], because the scalar field can diverge on the horizon and make the black hole become unstable [3]. In higher-dimensional cases, the hairy black hole solutions in asymptotically flat spacetimes simply do not exist [4, 5]. Considering the cosmological constant, it was expected that the no-hair theorem can be circumvented. Some hairy black holes were found in asymptotically de Sitter spacetimes [6–8], but they were proved dynamically unstable [9, 10]. The dawn of constructing a black hole with scalar hair broke on the horizon when a negative cosmological constant was taken into account. Until now, there have been a lot of black holes with scalar hair constructed in the anti-de Sitter (AdS) Einstein gravity [11–31] as well as the generalized gravity theory with higher curvature and

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negative cosmological constant [32–36].

Thermodynamically, AdS black holes with scalar hair are very interesting. It was found that there exist thermodynamical phase transitions between hairy black holes and black holes without scalar hair in AdS spacetimes. For instance, it was observed that a three-dimensional AdS black hole with a minimally coupled scalar field hair can transform to a BTZ black hole [37, 38]. Moreover, in four-dimensional AdS spacetimes, a second order phase transition was disclosed to occur between the black hole with a minimally coupled scalar field hair (Martinez-Troncoso-Zanelli (MTZ) black hole) and topological black hole with hyperbolic horizon (TBH) [13, 38]. Furthermore, phase transitions among charged TBH, charged MTZ black holes [17, 39] and other exact hairy black hole solutions [25, 31] in the four-dimensional AdS spacetimes were also uncovered.

In this work, we want to ask whether the intriguing thermodynamical relation between the hairy black hole and black hole without scalar hair in AdS spacetimes can be reflected in dynamical properties so that it can have some observational signatures to be detected. This is our main purpose here. Considering that quasinormal modes of dynamical perturbations are characteristic sounds of black holes, we expect that the black hole phase transitions can be imprinted in the dynamical perturbations in their surrounding geometries through frequencies and damping times of the oscillations. Recently a lot of discussions have been focused on this topic, and more and more evidences have been found between thermodynamical phase transitions and dynamical perturbations, see for example [40–47]. The deep relation between the dynamical perturbations and the Van der Waals like thermodynamical phase transitions in RN-AdS black holes has been further disclosed in [48].

It is of great interest to generalize the discussions and find more examples on the relation between the dynamical physical phenomenon and its corresponding thermodynamic properties. In this work, we will concentrate our attention on the rotating black hole solution in three-dimensional Einstein gravity with nonminimally coupled scalar field, which was recently reconstructed in [49]. It was argued that the rotating BTZ black hole phase is more thermodynamically preferred and there exists a nonvanishing probability for a rotating hairy black hole to transform into a rotating BTZ black hole. Can these thermodynamical properties have some observational signature in black hole dynamics? Now we will concentrate on this topic and disclose the fact that quasinormal modes (QNM) of dynamical perturbations around black holes can again be an effective probe of the black hole thermodynamical properties.

The paper is organized as follows. In Sec. II, we first review the thermodynamical stabilities for the rotating black hole with a nonminimally coupled scalar hair, and phase transition between this

rotating hairy black hole and rotating BTZ black hole in three-dimensional AdS spacetimes. Then we will disclose numerically that this phase transition can be reflected by the QNM frequencies of perturbations. In Sec. III, similar investigations will be extended to static black hole with a nonminimally coupled scalar hair and static BTZ black hole in three-dimensional AdS spacetimes. We complete the paper with conclusions in Sec. IV.

II. THREE DIMENSIONAL ROTATING BLACK HOLE WITH A NONMINIMALLY COUPLED SCALAR HAIR

A. Phase transitions between the rotating hairy and rotating BTZ black holes

Let us consider the following action for the Einstein gravity with nonminimally coupled scalar field and negative cosmological constant ($\Lambda = -\frac{1}{l^2}$)

$$\mathcal{I} = \frac{1}{2} \int d^3x \sqrt{-g} \left(R + \frac{2}{l^2} - g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - \frac{1}{8} R \phi^2 - 2V(\phi) \right), \quad (1)$$

where l is the AdS radius. With the scalar potential

$$V(\phi) = \frac{1}{512} \left(\frac{1}{l^2} + \mu \right) \phi^6 + \frac{\alpha^2 (\phi^6 - 40\phi^4 + 640\phi^2 - 4608) \phi^{10}}{512(\phi^2 - 8)^5}, \quad (2)$$

and the scalar field

$$\phi(r) = \pm \sqrt{\frac{8B}{r+B}}, \quad (3)$$

the action [Eq. (1)] admits the rotating hairy black hole solution [49]

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2(d\varphi + \Omega(r)dt)^2, \quad (4)$$

$$f(r) = -M \left(1 + \frac{2B}{3r} \right) + \frac{(3r+2B)^2 J^2}{36r^4} + \frac{r^2}{l^2}, \quad \Omega(r) = -\frac{(3r+2B)J}{6r^3}. \quad (5)$$

where the parameters M and J denote the mass and angular momentum of this black hole with $M = -3\mu B^2$ and $J = 6\alpha B^2$, respectively. The parameters μ , α and B are integral constants.

From $f(r_+) = 0$, the horizon radius r_+ can be expressed as $r_+ = B \times \theta$, where θ is related to the parameters μ , α and l [49]. Moreover, we have $[B] = [l] = L$, $[\mu] = L^{-2}$ and $[\alpha] = L^{-1}$ by performing the dimensional analysis. Then the mass, temperature, entropy, angular velocity and

free energy of rotating hairy black hole are given by [49]

$$M = \frac{J^2 l^2 (2 + 3\theta)^2 + 36r_+^4 \theta^2}{12l^2 r_+^2 \theta (2 + 3\theta)}, \quad S = \frac{4\pi\theta r_+}{1 + \theta}, \quad (6)$$

$$T = \frac{(1 + \theta) [36r_+^4 \theta^2 - J^2 l^2 (2 + 3\theta)^2]}{24\pi l^2 r_+^3 \theta^2 (2 + 3\theta)}, \quad \Omega_H = \frac{(3\theta + 2) J}{6\theta r_+^2}, \quad (7)$$

$$F = \frac{J^2 l^2 (2 + 3\theta)^2 - 12r_+^4 \theta^2}{4l^2 r_+^2 \theta (2 + 3\theta)}. \quad (8)$$

In the limit $\phi = 0$, the action [Eq. (1)] admits the rotating BTZ black hole solution [50]

$$ds^2 = -f(\rho)dt^2 + f(\rho)^{-1}d\rho^2 + \rho^2 (d\varphi + \Omega(\rho)dt)^2, \quad (9)$$

$$f(\rho) = -\hat{M} + \frac{\rho^2}{l^2} + \frac{\hat{J}^2}{4\rho^2}, \quad \Omega(\rho) = -\frac{\hat{J}}{2\rho^2}. \quad (10)$$

The thermodynamic quantities, such as the temperature, mass, entropy, angular velocity and free energy of rotating BTZ black hole are given by

$$\begin{aligned} \hat{M} &= \frac{\rho_+^2}{l^2} + \frac{\hat{J}^2}{4\rho_+^2}, \quad \hat{T} = \frac{4\rho_+^4 - \hat{J}^2 l^2}{8\pi l^2 \rho_+^3}, \quad \hat{S} = 4\pi\rho_+, \\ \hat{F} &= \frac{3\hat{J}^2 l^2 - 4\rho_+^4}{4l^2 \rho_+^2}, \quad \hat{\Omega}_H = \frac{\hat{J}}{2\rho_+^2}. \end{aligned} \quad (11)$$

By introducing the matchings for the temperature $T = \hat{T}$ and angular momentum $J = \hat{J}$

$$\frac{(1 + \theta) [36r_+^4 \theta^2 - J^2 l^2 (2 + 3\theta)^2]}{24\pi l^2 r_+^3 \theta^2 (2 + 3\theta)} = \frac{4\rho_+^4 - J^2 l^2}{8\pi l^2 \rho_+^3}, \quad (12)$$

the result presented in [49] shows that the rotating BTZ black hole always has smaller free energy which is a thermodynamically more preferred phase, and there exists a possible phase transition for this rotating hairy black hole to become rotating BTZ black hole, provided that there are some thermal fluctuations. In the next subsection, it is interesting to study whether the nonvanishing probability for the rotating hairy black hole transforming into the rotating BTZ black hole can be reflected in dynamical properties under the electromagnetic perturbation. We expect that it can provide deep insights on the stability of the background.

B. Quasinormal modes of rotating black holes

The electromagnetic perturbations are governed by Maxwell's equations

$$F^{\mu\nu}{}_{;\nu} = 0, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \quad (13)$$

As the background is circularly symmetric, it would be advisable to expand A_μ in 3-dimensional vector spherical harmonics

$$A_\mu(t, r, \varphi) = \begin{pmatrix} P(r) \\ S(r) \\ Q(r) \end{pmatrix} e^{-i\omega t + im\varphi}, \quad (14)$$

where m is our angular quantum number and ω is the frequency.

Considering the metric [Eq. (4)], we can substitute the expansion [Eq. (14)] into Maxwell's equations [Eq. (13)] and obtain

$$\begin{cases} F^{t\nu}_{;\nu} = \frac{d}{dr} (rH(r)\Omega(r)) + \frac{d}{dr} (rK(r)) - \frac{m(mP(r)+\omega Q(r))}{rf(r)} = 0, \\ F^{r\nu}_{;\nu} = -mr^2H(r)\Omega(r) + (mf(r) - \omega r^2\Omega(r))H(r) - (\omega + m\Omega(r))r^2K(r) = 0, \\ F^{\varphi\nu}_{;\nu} = \frac{d}{dr} (rH(r)\Omega(r)^2) + \frac{d}{dr} (rK(r)\Omega(r)) - \frac{d}{dr} \left(\frac{f(r)H(r)}{r} \right) + \frac{\omega r(mP(r)+\omega Q(r))}{r^2f(r)} = 0, \end{cases}$$

where

$$H(r) = imS(r) - \frac{dQ(r)}{dr}, \quad K(r) = i\omega S(r) + \frac{dP(r)}{dr}. \quad (15)$$

Based on these equations, a second order differential equation for the electromagnetic perturbation can be derived as

$$\psi''(r) + \frac{f'(r)\psi'(r)}{f(r)} + \left[\frac{(m\Omega(r) + \omega)^2}{f^2(r)} + \frac{1}{4r^2} - \frac{m^2}{r^2f(r)} - \frac{f'(r)}{2rf(r)} \right] \psi(r) = 0, \quad (16)$$

where the wavefunction $\psi(r)$ is set to be a combination of functions $f(r)$, $\Omega(r)$, $S(r)$ and $Q(r)$

$$\psi(r) = \frac{f(r)}{r^{1/2}} \frac{\omega}{m\Omega(r) + \omega} \left(imS(r) - \frac{dQ(r)}{dr} \right).$$

Adopting the tortoise coordinate r_* (defined by $dr_* = \frac{dr}{f(r)}$), Eq. (16) becomes

$$\begin{aligned} \partial_*^2 \psi(r) + \left[(m\Omega(r) + \omega)^2 - V(r) \right] \psi(r) &= 0, \\ V(r) &= \frac{f(r)(4m^2 - f(r))}{4r^2} + \frac{f(r)f'(r)}{2r}. \end{aligned} \quad (17)$$

Now we can evaluate the appropriate boundary conditions. At infinity, we have $\psi(r) = 0$ because of $V(r) \rightarrow +\infty$. Near the horizon, the incoming wavefunction $\psi(r)$ reads as

$$\psi(r) \sim (r - r_+)^{-\frac{i\tilde{\omega}}{4\pi T}}, \quad \tilde{\omega} = \omega - \frac{(3\theta + 2)JM}{6\theta r_+^2}. \quad (18)$$

Here we can define $\psi(r)$ as $\psi(r)\exp[-i\int \frac{\tilde{\omega}}{f(r)}dr]$, where $\exp[-i\int \frac{\tilde{\omega}}{f(r)}dr]$ asymptotically approaches to the ingoing wave near horizon, and then the Eq. (16) becomes

$$\begin{aligned} &4r^2f(r)\psi''(r) + 4r^2\psi'(r)[f'(r) - 2i\tilde{\omega}] \\ &+ \left[f(r) - 2(2m^2 + rf'(r)) + \frac{4r^2((m\Omega(r) + \omega)^2 - \tilde{\omega}^2)}{f(r)} \right] \psi(r) = 0 \end{aligned} \quad (19)$$

so that $\psi(r_+) = 1$ in case of $r \rightarrow r_+$. Then we can numerically solve Eq. (19) and find the QNM frequencies under the boundary conditions by adopting the shooting method.

Thermodynamical stabilities can be disclosed by examining the free energy. The free energies of the rotating hairy black hole with different values of θ and angular momentum J are plotted in FIG. 1. We see that the free energy F becomes more negative for smaller value of J or larger value of θ to ensure that the corresponding hairy black hole becomes more thermodynamically stable. Fixing the angular momentum, there appears a degeneracy in the free energy when θ is big enough.

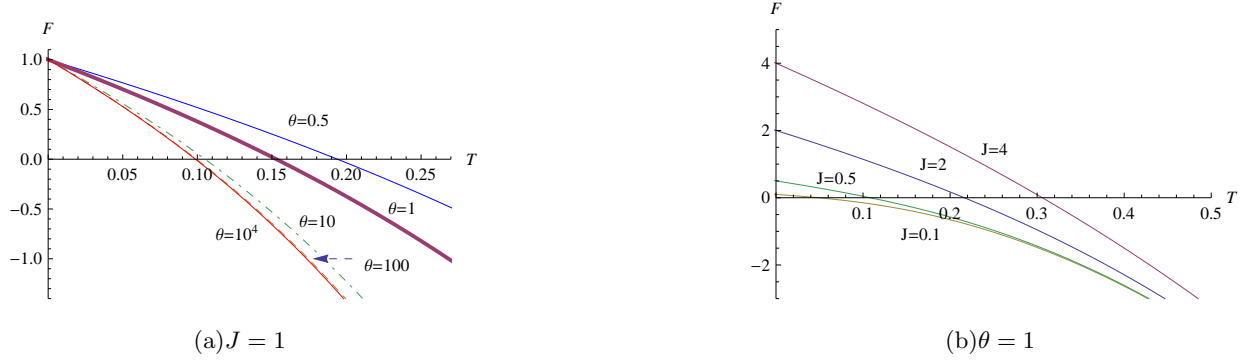


FIG. 1: The free energies F of rotating hairy black hole versus the temperature T with $l = 1$ in three-dimensional AdS spacetimes.

To reflect the thermodynamical properties in dynamical perturbations, we list the numerical values of the lowest frequencies of perturbations around this rotating hairy black hole for taking $m = 1$ at $T = 0.5$ in Table I. One can see that the imaginary part of QNM frequencies becomes more negative with the increase of θ , or the decrease of J . When the value of θ becomes big enough, the imaginary part of QNM frequency approximately converges to $-5.921i$ and will not change. The dynamical result shows that the rotating hairy black hole is more stable for larger θ or smaller J . This exactly reflects the thermodynamic stability of the rotating hairy black hole disclosed in FIG. 1.

In [49], after comparing the rotating hairy black hole and the rotating bald BTZ black hole, it was argued that the rotating bald BTZ black hole phase is more thermodynamically preferred, and there exists a nonvanishing probability for the rotating hairy black hole to transform into rotating bald BTZ black hole. The argument was based on the thermodynamical analysis of the free energies F as we replotted FIG. 2 where the solid lines are for the rotating bald BTZ holes and dashed lines are for rotating hairy black holes.

In order to confirm the thermodynamical argument in dynamics, we numerically calculate the QNM frequencies of rotating bald BTZ black hole by replacing $f(r)$ with $f(\rho)$ in Eq. (19). The

rotating hairy black hole($J = 1$)		
θ	r_+	ω
0.5	2.52774	2.54513-4.93277I
1	2.65510	2.37348-5.22896I
10	3.05636	1.29700-5.89554I
100	3.13941	1.02824-5.91643I
10^4	3.14949	0.99079-5.92057I
10^6	3.14959	0.99040-5.92061I

rotating hairy black hole($\theta = 1$)		
J	r_+	ω
10	3.84226	1.17714-4.09625I
4	3.02100	1.58439-4.81141I
2	2.75136	2.13435-5.12212I
0.5	2.62756	2.47978-5.39292I
0.1	2.61838	2.58133-5.51261I
0.05	2.61809	2.59570-5.52749I

TABLE I: Lowest QNM frequencies of electromagnetic perturbations with $m = 1$ for the rotating hairy black hole at $T = 0.5$.

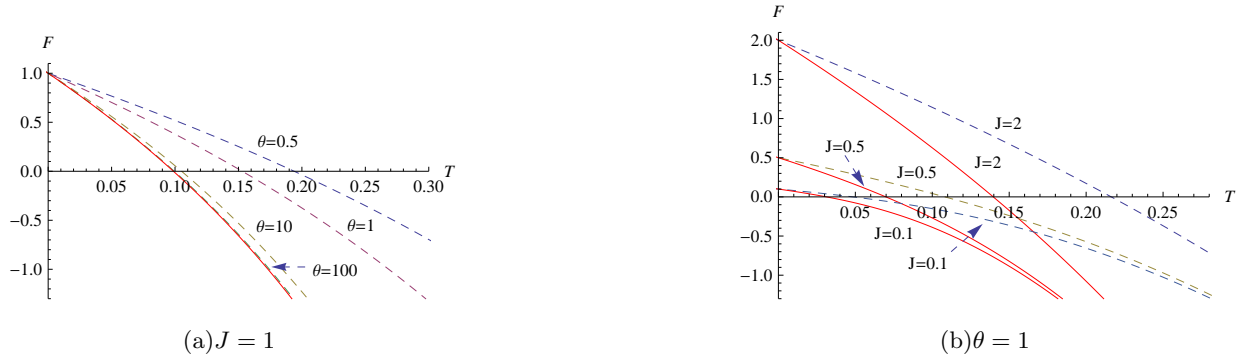


FIG. 2: The free energies F of rotating hairy black hole (dashed lines) and rotating BTZ black hole (solid lines) versus the temperature T with $l = 1$ in three-dimensional AdS spacetimes.

numerical results of the lowest QNM frequencies of rotating bald BTZ black hole are displayed in Table.II where we have taken $m = 1$. Analytic expression of the QNM frequency of rotating BTZ black hole can be obtained as [51]

$$\omega = m - 2i(M - J)^{1/2}(n + 1), \quad n = 0, 1, 2, \dots \quad (20)$$

Taking $m = 1$ and $n = 0$, we find that our numerical results agree excellently with the analytical values of QNM frequencies for the rotating BTZ black hole. Comparing with the QNM frequencies of rotating hairy black hole, we find that although the imaginary part of QNM frequencies for the hairy hole becomes more negative with the growth of θ , it is always larger than that of the rotating bald BTZ black hole at any temperature, see Table. II. This shows that the rotating bald BTZ black hole is dynamically more stable, which agrees exactly with the thermodynamical stability analysis. The result also tells us that there exists a nonvanishing possibility for the rotating hairy black hole to decay into the rotating bald BTZ black hole.

	rotating hairy black hole						rotating BTZ black hole		
T	r_+	$\omega^{Num}(\theta = 1)$	r_+	$\omega^{Num}(\theta = 10)$	r_+	$\omega^{Num}(\theta = 10^4)$	r_+	ω^{Num}	ω^{Ana}
0.1	1.07798	1.05400-0.70966I	0.94565	1.01481-0.77984I	0.93459	1.00001-0.79911I	0.9346	1.00000-0.79916I	1.0-0.79915I
0.3	1.70974	1.49730-2.83011I	1.87125	1.14435-3.25943I	1.92021	1.00105-3.28684I	1.92026	1.00478-3.32504I	1.0-3.31980I
0.5	2.65510	2.37348-5.22896I	3.05636	1.29700-5.89554I	3.14949	0.99079-5.92057I	3.14959	0.96490-5.93929I	1.0-5.98168I
0.8	4.19818	3.68705-8.67322I	4.87668	1.66840-9.65182I	5.02835	1.08189-9.77114I	5.02851	0.94564-9.78255I	1.0-9.85815I
1.0	5.24081	4.56048-10.91802I	6.09404	1.98665-12.09234I	6.28398	1.11556-12.31122I	6.28419	1.12832-12.33469I	1.0-12.40930I

TABLE II: Lowest QNM frequencies of electromagnetic perturbations with $m = 1$ of black holes for $J = 1$.

III. THREE DIMENSIONAL STATIC BLACK HOLE WITH A NONMINIMALLY COUPLED SCALAR HAIR

A. Phase transitions between the static hairy black hole and BTZ black hole

Now, we turn to study the static black hole with nonminimally coupled scalar field in three-dimensional AdS spacetimes. Setting $\alpha = 0$, namely $J = 0$, the scalar potential becomes

$$V(\phi) = \frac{1}{512} \left(\frac{1}{l^2} + \mu \right) \phi^6, \quad (21)$$

and the static hairy black hole solution is

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2d\varphi^2, \quad (22)$$

$$f(r) = -M \left(1 + \frac{2B}{3r} \right) + \frac{r^2}{l^2}, \quad M = -3\mu B^2. \quad (23)$$

Then the mass, entropy, temperature and free energy of this hairy black hole are

$$M = \frac{3r_+^2 \tilde{\theta}}{(2 + 3\tilde{\theta})l^2}, \quad T = \frac{3(1 + \tilde{\theta})r_+}{2\pi l^2 (2 + 3\tilde{\theta})}, \quad S = \frac{4\pi \tilde{\theta} r_+}{1 + \tilde{\theta}}, \quad (24)$$

$$F = -\frac{\tilde{\theta}(2 + 3\tilde{\theta})}{3(1 + \tilde{\theta})^2} (2\pi l T)^2. \quad (25)$$

Here the black hole radius can be also written as $r_+ = B \times \tilde{\theta}(\mu, l)$ with $\tilde{\theta}(\mu, l) > 0$ [18].

For $\phi = 0$, the static BTZ black hole reads as [50]

$$ds^2 = -f(\rho)dt^2 + f(\rho)^{-1}d\rho^2 + \rho^2d\varphi^2, \quad f(\rho) = -\hat{M} + \frac{\rho^2}{l^2}, \quad (26)$$

and then the temperature \hat{T} , mass \hat{M} , entropy \hat{S} and free energy \hat{F} of the BTZ black hole are given by

$$\hat{M} = \frac{\rho_+^2}{l^2}, \quad \hat{T} = \frac{\rho_+}{2\pi l^2}, \quad \hat{S} = 4\pi\rho_+, \quad \hat{F} = -(2\pi l \hat{T})^2. \quad (27)$$

Now considering the thermal equilibrium by equating the temperature for the static BTZ hole and static hairy black hole

$$\frac{3(1+\tilde{\theta})r_+}{2\pi l^2(2+3\tilde{\theta})} = \frac{\rho_+}{2\pi l^2}, \quad (28)$$

we can examine the differences of these two black holes in their free energies

$$\Delta F = F - \hat{F} = \frac{3+4\tilde{\theta}}{3(1+\tilde{\theta})^2}(2\pi lT)^2. \quad (29)$$

Evidently, we have $\Delta F > 0$, namely $F > \hat{F}$ because of $\tilde{\theta} > 0$. In this case, the BTZ black hole is more thermodynamically preferred, and there is a nonvanishing probability for decay of the black hole with nonminimally coupled field into BTZ black hole. Moreover, ΔF approaches zero when $\tilde{\theta} \rightarrow +\infty$.

B. Quasinormal modes of static black holes

We turn to examine the differences of these two black holes in dynamics. Setting $\Omega(r) = 0$, the equation of electromagnetic perturbations for the static metric [Eq. (19)] reduces to

$$\psi''(r) + \left[\frac{f'(r)}{f(r)} - \frac{2i\omega}{f(r)} \right] \psi'(r) + \left[\frac{1}{4rf(r)} - \frac{m^2}{r^2f(r)} - \frac{f'(r)}{2rf(r)} \right] \psi(r) = 0. \quad (30)$$

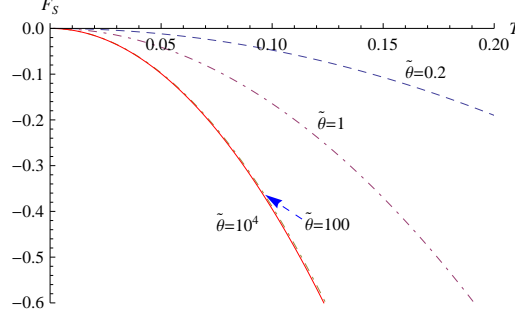
Similarly, we have the boundary conditions: $\psi(r_+) = 1$ when $r \rightarrow r_+$ and $\psi(r) = 0$ when $r \rightarrow +\infty$.

From thermodynamics, we see that the free energy F of the hairy black hole becomes more negative with the growth of $\tilde{\theta}$ and degenerates when $\tilde{\theta}$ is large enough, see FIG. 3, which tells us that the hairy black hole with larger $\tilde{\theta}$ is thermodynamically more stable. When $\tilde{\theta} \rightarrow +\infty$, the free energy F approaches $-(2\pi lT)^2$.

From the dynamical perturbation analysis, we have the lowest frequencies of QNM for the hairy black hole listed in TABLE. III, where we have taken $m = 1$ and $T = 0.5$. We observe that the imaginary part of QNM frequencies becomes more negative with the increase of $\tilde{\theta}$, and converges to -6.13I when $\tilde{\theta}$ is big enough. The dynamical property uncovered from the perturbations around the hairy black hole exactly supports the thermodynamic stability result obtained for this hairy black hole.

In order to compare with the bald BTZ black hole, we calculate the frequencies of its QNM. The analytic result is [52]

$$\omega = m - 2iM^{1/2}(n+1), \quad n = 0, 1, 2, \dots \quad (31)$$

FIG. 3: Free energies of hairy black hole with $l = 1$.

hairy black hole		
$\tilde{\theta}$	r_+	ω
0.2	2.26893	3.05431-4.80924I
1	2.61799	2.61025-5.54201I
10	3.04639	1.42126-5.96734I
100	3.13122	0.96830-6.11964I
10^4	3.14149	0.94192-6.12771I
10^6	3.14159	0.94164-6.12779I

TABLE III: Lowest QNM frequencies of electromagnetic perturbations for $m = 1$ for the hairy black hole at $T = 0.5$.

The numerical result of the lowest QNM frequency and its comparison with the analytic results for taking $m = 1$ are shown in TABLE. IV. For any certain temperature, we find that the imaginary part of QNM frequencies of BTZ black hole is always smaller than that of the hairy black hole. This shows that the BTZ black hole is dynamically more stable, which agrees well with the study of the thermodynamical stability. Hence the properties disclosed here in electromagnetic perturbations reflect the phase transition of the decay from the hairy black hole into BTZ black hole.

T	hairy black hole						BTZ black hole		
	r_+	$\omega^{Num}(\tilde{\theta} = 1)$	r_+	$\omega^{Num}(\tilde{\theta} = 10)$	r_+	$\omega^{Num}(\tilde{\theta} = 10^4)$	r_+	ω^{Num}	ω^{Ana}
0.1	0.52360	1.16811-1.01974I	0.60928	1.07981-1.15040I	0.62830	0.96493-1.17736I	0.62832	1.03037-1.18224I	1.0-1.25664I
0.3	1.57080	1.76653-3.24229I	1.82784	1.20281-3.58872I	1.88489	1.04374-3.65702I	1.88496	1.11939-3.67200I	1.0-3.76992I
0.5	2.61799	2.61025-5.54201I	3.04639	1.42126-5.96734I	3.14149	0.94192-6.12771I	3.14159	0.93811-6.28258I	1.0-6.28318I
0.8	4.18879	3.37366-8.79366I	4.87423	1.63976-9.72062I	5.02638	1.13483-9.91516I	5.02655	0.97588-9.91772I	1.0-10.05310I
1.0	5.23599	4.59046-11.00070I	6.09279	1.97289-12.13969I	6.28298	1.18441-12.43421I	6.28319	1.04346-12.53968I	1.0-12.56640I

TABLE IV: Lowest QNM frequencies of electromagnetic perturbations of black holes for $m = 1$.

IV. CONCLUSIONS

We have calculated the QNMs of electromagnetic perturbations for the rotating and static three-dimensional AdS black holes with nonminimally coupled scalar field and compared with the dynamic properties of their corresponding BTZ black hole counterparts. We have found that the imaginary part of quasinormal frequencies of rotating hairy black hole becomes more negative with the increase of its parameter θ or the decrease of the angular momentum J . These results tell us that rotating hairy black holes with bigger θ and smaller J can be more dynamically stable objects. The disclosed dynamical stability properties are consistent with the thermodynamical stability.

Comparing with the corresponding rotating BTZ black hole counterparts, we have observed that the rotating black holes always have bigger absolute imaginary QNM frequencies. This dynamical property holds at any temperature. This tells us that comparing with the rotating hairy black hole, the rotating BTZ black hole counterpart is dynamically more stable. This property is again in agreement with that disclosed thermodynamically that the rotating BTZ black hole has lower free energy which is more thermodynamically preferred. Similar observations on the dynamical phenomenon and its relation to thermodynamics have also been disclosed between the three-dimensional AdS static hairy hole and its BTZ black hole counterpart.

Here we have obtained two more examples to show that the QNM can provide physical phenomenon in dynamics to support the disclosed thermodynamic properties. Considering that the QNM is expected to be detected and has strong astrophysical interest, we hope that the dynamical probe can really present us the observational signature of the thermodynamic property of black holes.

Acknowledgments

This work was supported by the National Natural Science Foundation of China.

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